A New Approach to Inertial Precision Positioning and Navigation without Accelerometers

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ABSTRACT

Based upon extending the Sagnac effect, i.e. the inertial Doppler effect $v/c = \Delta s/L$, from rotational to linear motion the paper outlines the concept of digitally measuring three-dimensional vehicle velocities, with earth as the reference, and derive 3D-position at any time by velocity integration over time. Vehicle attitude control can be achieved by employing the same principle of digital inertial velocity determination to also measure tangential or rotational velocities. Such devices are suitable to determine north and plumb, too.

INTRODUCTION

Inertial positioning and navigation of professional aircraft is based upon real or strapdown three-axis platforms and accelerometers for each axis. Continuous integration of accelerations twice over time leads to the respective travelled distances in each axis, referred to any selected known starting point of any mission. While this technology has been matured over many decades, substantial reductions of errors, drift and cost are still being desired. Therefore TIP -Time-based Inertial Positioning - a new approach to inertial positioning and navigation is outlined, which does require neither accelerometers nor conventional platforms. This approach is based upon inertial velocity measurements which are performed by special digital sensors. These v-sensors utilise the first-order inertial Doppler effect related to utilising electromagnetic signals propagating with light velocity c within matter moving at a velocity v with respect to c.

THE INERTIAL DOPPLER EFFECT

The Inertial Doppler Effect IDE is outlined in fig. 1a-c, using for demonstration a radio link comprising an earth station T-R and a geostationary satellite GS, the distance of which to earth surface is roughly 3.10^7 m. Orbit velocity of earth is 3.10^4 ms^{-1}. The assumed configuration involves the transmitted signal having a propagation vector in line with the earth orbit velocity vector **v**, while the propagation vector of the signal reflected by GS is in opposition to **v**. At the first glance one might assume that there is no first order Doppler effect since there seems to be no relative motion between T, R, and GS.

However, this is not true, as is outlined in diagram 1b. There it is shown how the radio frequency signal transmitted to and returning from GS uses a propagation path with a length L, while the direct path from T to R be so short that its length can be neglected compared to L. If the two signals, the direct one and the one via L, are conventionally mixed with each other in mixer M, an IDE results, which represents the velocity v of the system, namely 30 kms^{-1} .

Those who think that there is no relative motion between the three components T, R, and GS, do not fully err, of course, but are half right instead. What happens is the following. The propagation path L/2 from T to GS is extended by the small path element Δs, since GS is moving with v while the transmitted signal has to propagate with c through L/2. Thus the total path travelled is L/2 + Δs. For the reflected signal travelling from GS to R, the propagation path is shortened by Δs, due to the opposition of both vectors **c** and **v**, resulting in a total travel path of L/2 - Δs. If one *adds* both travelling paths of different lengths, one gets L, and no IDE results. If, however, one *subtracts* both paths from each other, one faces a differential path of 2Δs. These elements are the spatial distance between T at the time of transmission, and R at the time of reception. Thus there is relative motion between T and R, seen inertially, and therefore the IDE results. On the other hand, if the system is viewed by an observer who is travelling with the system, i.e. who is moving with v, no first order Doppler effect results. The Relative Doppler Effect RDE is zero. This is what we as observers on the moving earth normally notice, since the IDE is usually very small. In summary, the configuration shown in fig 1a has no RDE, but an IDE, the magnitude of which is shown in fig.1c, provided subtraction instead of summation of both signals is performed. If one uses a transmitter clock rate of 1 Gbit/s, the clock rate of the received signal is reduced by 20 000 bit during the propagation time of 0.2 s, and by 100 000 bit when measuring for a second, corresponding to $(v/c)\ f_0 = 10^{-4}\ f_0$. Such Doppler effects are easy to measure, even during college exercises.

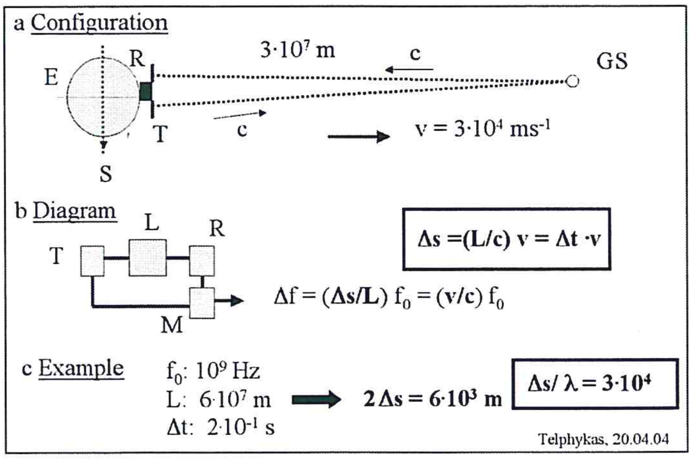

Figure 1: Inertial Doppler Effect

FROM "ROTATIONAL" TO "LINEAR" INERTIAL VELOCITY MEASUREMENT

Principally it makes little sense to discriminate between rotational and linear velocities on earth since there is no real discrimination between both. What is called "linear" on earth surface, is in fact rotational with radius r being $6{,}378.10^6$ m if the earth is taken as reference. And what we might call rotational, when r is only 10^{-1} m, might be considered as "linear", if a reference length is given in micrometer. If the sun is taken as reference, then the orbit motion of earth appears, of course, to be rather linear during short periods of time, due to its high orbit velocity and distance to sun of about 150 million kilometers.

The application of the Sagnac effect is well established for measurements of rotation rates by ring lasers or fibre "gyros". In fig. 2 the principle is shown. The propagation path $L = 2\pi r$ is changed by a very small path element Δs. For clockwise propagation and rotation the propagation path is $L + \Delta s$, for counter clockwise propagation this path is $L - \Delta s$, which leads to the path difference $2\Delta s$ and a Doppler effect $2\Delta s/L = 2v_T/c$. It can be measured as a Doppler frequency $2\Delta f = 2(v_T/c)\, f_0$. Usually f_0 is an optical frequency in the range $10^{14} – 10^{15}$ Hz. This is due to the very small values of v_T when earth rate $7{,}27.10^{-5}\ s^{-1}$ is involved. Again one may consider Δs as the result of motion of receiver R with v_T for the time duration L/c which light starting at T requires to reach R. Δs is an inertial distance between transmitter and receiver, and influenced by several parameters as will be outlined in a later section.

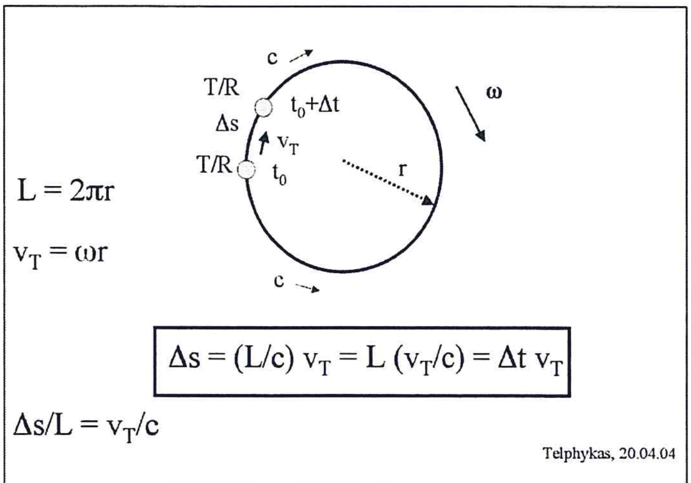

Figure 2: Sagnac Effect

The principle shown in fig. 1 and fig. 2 can also be employed for small linear devices as is shown in fig 3. A signal source T feeds, in opposite directions, two propagation paths, which move with velocity v. Receiver R1 moves by Δs to the left of

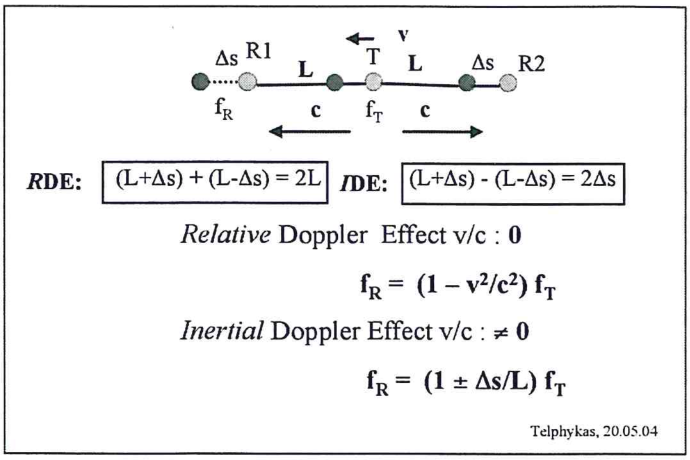

Figure 3: Physical Base v/c=Δs/L

the sketch, thus extending the propagation path L to $L + \Delta s$. Receiver R2 also moves to the left, thus shortening the respective propagation path L to $L - \Delta s$. There is no RDE when both propagation paths are added, since Δs is eliminated then and no relative motion between T and R1, R2 results. If, however, both paths are subtracted from each other, twice the path element Δs remains, which results in a first-order inertial Doppler effect, IDE, while the first-order relative Doppler effect RDE is zero, and only the second-order Doppler effect v^2/c^2 could be detected in this case. The IDE can be used to measure "linear" velocities on earth. "Linear" always means an approximation since all movements on earth are eventually circular motions and not exactly linear.

DIGITAL INERTIAL VELOCITY MEASUREMENT

The principal of a digital inertial velocity sensor is sketched in fig. 4. Such inertial velocity sensor comprises two digital high-speed counters 3 and 4 which are fed by a signal source 1 in line and, respectively, in opposition to motion direction, via travel path extensions, or delay lines, 2 and 4 having the length L each. The numerical difference of both counters during each counting period, measured by a subtracting device 6, is proportional to motion velocity v of the sensor within the gravity field of the sun.

An inertial velocity measurement is achieved by the pair of digital counters 3-5 and the pair of delay lines 2-4 fed by a clock 1 with a GHz frequency (e.g. 3 GHz). One counter "sees" velocity and signal propagation vectors having the same direction, the other counter "sees" them as being opposite to each other. At $v \neq 0$ (which is always given on earth!) one counter receives an increased clock frequency $(1+v/c)f_0$, while the other counter receives a reduced frequency $(1-v/c)f_0$. The frequency difference of both counters, measured by subtractor 6, leads to a numerical value which is proportional to $2(v/c)f_0$. This value does also contain the (known) velocity of the respective sensor position due to orbit and rotational velocities of earth within the gravity field of the sun. By computing consideration of these one receives the velocity of the sensor with respect to earth surface.

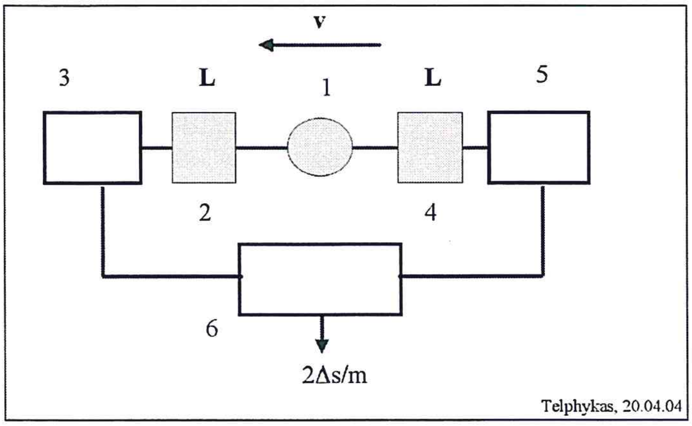

Figure 4: Digital Inertial v-Sensor

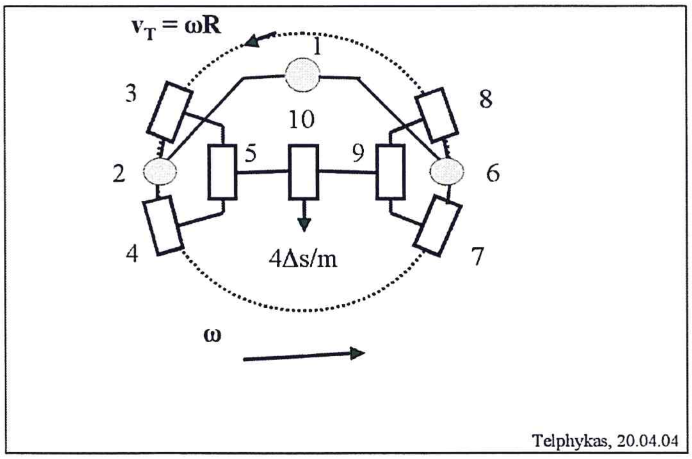

Figure 5: Digital Inertial ω-Sensor

In fig. 5 a digital inertial rotation rate sensor (ω-sensor) is sketched. It raises a key question: How can this sensor measure rate when its basic elements, e.g. oscillator 1, signal splitter 2, counters 3 and 4, and subtractor 5 look identical to the respective elements of a v – sensor? The bits at the output of the subtractor 5 would represent a differential velocity. This observation is, of course, correct. A rate sensor resting on earth surface would move inertially along with the orbit velocity plus or minus the respective rotational velocity of earth. The bits at the outputs of subtractors 5 and 9 would therefore represent orbit velocity and daily rotation rate of earth, however, with a slight difference. This difference is due to the different motion directions, seen inertially, of the one sensor element and the opposite other one on the circle, when the sensor rotates with respect to earth. This approach is different from the classic Sagnac effect which is applied in ringlasers and fiber-optic rate sensors to propagation paths involving an area, a configuration which automatically eliminates orbit and daily rotation velocities of earth. A quantitative analysis indicates the potential performance of this alternative approach to Sagnac. With an orbit velocity of $v_O = 3.10^4$ ms^{-1}, a rotational velocity of $v_R = 467$ ms^{-1} at the equator, and a rotational velocity of the rate sensor of $v_T = 10^{-1}$ ms^{-1}, the measured velocities at the subtractors of both sensors would be $v_{34} = v_O^* + v_R^* + v_T$, and $v_{78} = v_O^* + v_R^* - v_T$. If these values are subtracted from each other by subtractor 10, one receives $v_{34} - v_{78} = 2\, v_T$. This equals 2.10^{-1} ms^{-1}. In order to measure this, the velocity resolution of the sensor would have to be in the order of 10^{-2} ms^{-1} (even 10^{-3} ms^{-1}), which can be achieved without major problems since it would represent a ratio of $3{,}33.10^{-6}$. A different story is to measure the daily earth rate via this approach. In this case v_T is smaller by four orders of magnitude in the range of 10^{-10} ms^{-1}. In any case, this approach is not subject to the lock-in effect of traditional Sagnac-based rate sensors.

In fig. 6 numerical examples for a velocity sensor and a rate sensor are given for assumed dimensions. With a clock rate of 3.10^9 s^{-1} and L = 3,333 m for a v-sensor one gets a velocity resolution of 0,15 m s^{-1} and a difference of 10 between the counters, a figure, which represents a position resolution of 0,15 m. For the rate sensor L = 3333 m is assumed, but despite an increase of L by a factor of 10^3 the measurement of the earth rate requires a number of seconds, since in one second a bit difference of only 0,134 would be generated.

The very small tangential velocity v_T of a rate sensor, compared to the orbit velocity of earth suggests to look for approaches to increase the sensitivity of digital inertial rate sensors: As is shown in fig. 7, there are a least four approaches to increase the scale factor of such a rate sensor. At first, one can increase the clock rate. If one goes from a clock rate of 10^9 s^{-1} to a clock rate of 10^{10} s^{-1} one gains a factor of ten. If one increases also the length of L by a factor of ten one gains another factor 10. If one operates ten counters in parallel at one delay line L, another factor of 10 is gained. If all three measures are taken, a factor of 10^3 is the result. Of course, these measures would also apply to a velocity sensor. Now, finally, also the integration time can be extended. If ten seconds would be used instead of one second for integration, then a further factor of 10 would be gained.

a <u>v-Sensor</u>

$f_0 = 3\cdot10^9$ s^{-1}
$L = 3{,}333$ m
$v_0 = 300$ ms^{-1}
$v_B = 30\,000$ ms^{-1}
$(v_B + v_0)/c = 10^{-4}$
$\Delta L = (v_B + v_0/c)\, L = 3{,}333\cdot10^{-4}$ m
$\Delta\lambda = 0{,}333\cdot10^{-9}$ m

$2\Delta L/\Delta\lambda = 2000\,000$ s^{-1}

0,15 ms^{-1} : ≈ 10 s^{-1}

b <u>ω - Sensor</u>

$f_0 = 3\cdot10^9$ s^{-1}
$L = 3{,}333\cdot10^3$ m
$v_{RE} = R\omega = 10^{-6}$ ms^{-1}
$v_{RE}/c = 3.333\cdot10^{-15}$
$\Delta L = (v_{RE}/c)\, L = 1{,}111\cdot10^{-11}$ m
$\Delta\lambda = 0{,}333\cdot10^{-9}$ m

$4\Delta L/\Delta\lambda = 0{,}134$ s^{-1}

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Figure 6: Numerical Examples

3D-POSITIONING

The basic structure of TIP for an aircraft comprises a strapdown 3-axis-system to determine partial velocities orthogonal to each other, three orthogonal rate sensors for attitude control and two rate sensors providing north and plumb. The v-sensors, the rate sensors and both north and plumb sensors feed the computer which computes position, velocities and attitude of the measuring vehicle.

Combining three of these orthogonal v-sensors in a strapdown configuration delivers the three coordinates x, y and z, (or longitude, latitude and altitude respectively) while the three rotational values yaw y, pitch p and roll r, required additionally for attitude determination are delivered by

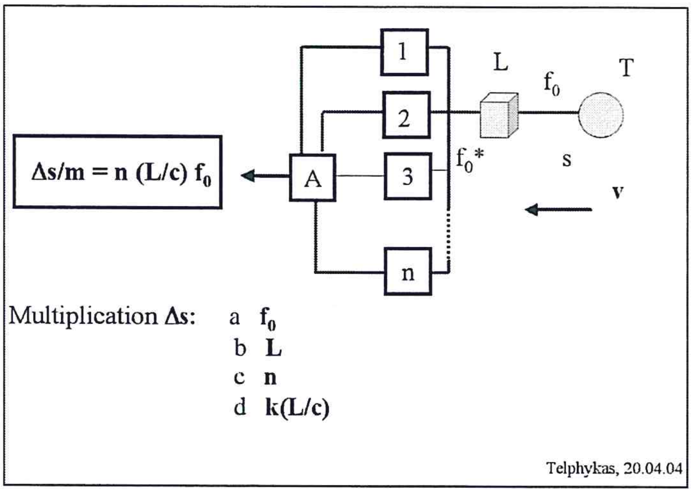

Figure 7: Effect Multiplication

another triple of orthogonal v-sensors separated from the first triple by a small distance of some centimeters. In addition, there are a north and a plumb sensor. The block diagram for 3D positioning is shown in fig.8.

The block diagram of TIP comprises three velocimeters v_m, v_p and v_v for measuring forward, side and vertical velocities, three attitude sensors, north and plumb sensors, computer SC, interface module I and power module P. All modules are linked via a bus.

Of course, there are many configurations possible, corresponding to numerous applications different from each other. Therefore discussion of these properties is skipped here. With respect to early future applications micro air vehicles may evolve as promising platforms for TIP.

NORTH AND PLUMB DETERMINATION

Determination of geographic north and also plumb is well known from optical rate sensors. The criterion for north is maximum output of rate sensor the axis of which is pointing horizontally along a parallel, i.e. the rate sensor plane is oriented vertically and along a meridian.

If a v-sensor is to be used for north finding orientation of this device to the vector $\mathbf{v_R}$ is to be perpendicular as sketched in fig.9a. The criterion for north is $v_R = 0$. In this case the *horizontal* v-sensor points north. Of course, the sensor measures some component of the orbit velocity of earth. But since this is known it can be subtracted from the measured signals and enable TIP to find the angle for which the criterion $v_R = 0$ is valid. A variety would be to use a horizontal v-sensor oriented along the local parallel. The (not very sharp) criterion for north would then be v_R = max.

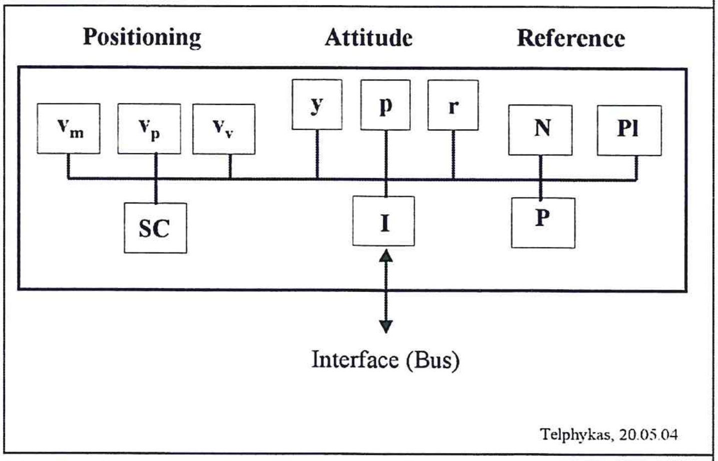

Figure 8: Block Diagram and Function of TIP

Determination of the plumb vector achieved not via gravity, i.e. dynamic criteria, but via kinematic ones is sketched in fig. 9b. The plumb vector at any point on earth is also characterised by the criterion: $v_R = 0$. This is sketched in fig. 9b. A v-sensor oriented *vertically* would be unable to measure the local rotational velocity v_R of earth. It would , however, measure some component of the orbit velocity proportional to its location and orientation, $v_B = v_O \cos \alpha \cos \phi$. α is the inertial azimuth, while ϕ is the inertial elevation, referred to the ecliptic. But this is known and can be considered properly, like in the case of north finding.

ANTICIPATED GENERIC SYSTEM CHARACTERISTICS

There is a very wide range of applications which can't be met by just one TIP version. Therefore only kind of a generic characteristics profile should be given here as an example. Fig.10 shows a selection anticipated as generic system characteristics. Emphasis is put upon the achievable

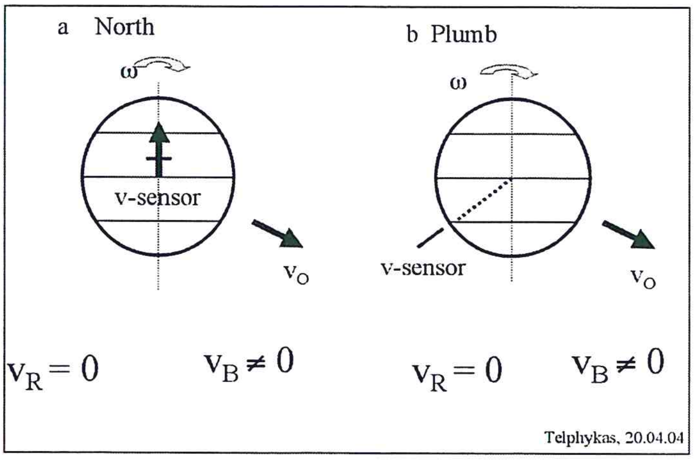

Figure 9: North and Plumb Vectors

positioning accuracy, a 0.5 m (3σ) error, which via using a radio system in a complex propagation environment might not be achievable at all.

COMPARISION OF TIP TO EXISTING INERTIAL POSITIONING AND NAVIGATION SYSTEMS

When a new approach to inertial positioning and navigation is proposed it should be made clear what the major advantages would be. Of course, the key issue is always the optimisation of performance to cost ratio. It is difficult, however, to compare such available ratios of existing systems to promised ratios of new systems not yet existing. Therefore one has to rely upon inherent factual advantages of a propagated future system. This is done in fig. 11, which for positioning compares a v-sensors approach with inertial velocity measurements and an a-sensor approach involving acceleration measurements.

As a common base for both approaches five different tolerable metric errors are assumed, from 0,1 m to 1000 m, for a ten hours mission. These errors require to provide

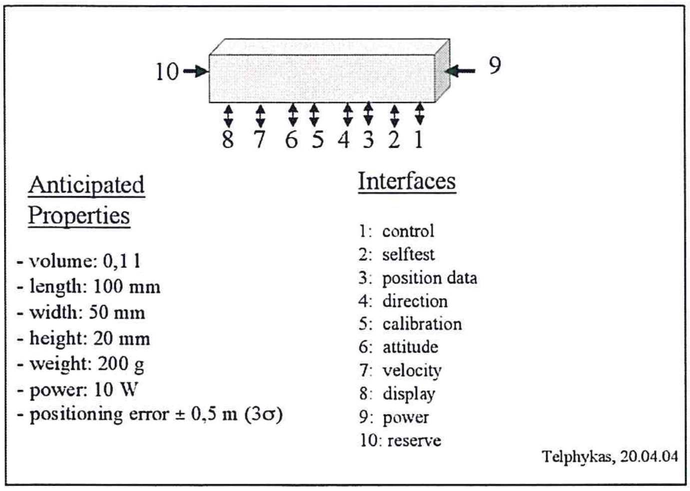

Figure 10: Anticipated Properties

sensors with respective sensitivities. In the case of v-sensors sensitivities can be reduced by four orders of magnitude compared to application of a-sensors. This has nothing to do with technologies or ingenuity, but is due to the equation $x = vt = at^2/2$. The elimination of one level of integration over time brings about the apparent dramatic advantage of v-sensors over a-sensors. There may be numerous details where accelerometers can claim pros. But the basic con is just the considerable lack of precision of accelerometers, comparatively.

COMPARISION OF TIP TO SATELLITE-BASED POSITIONING

A further comparison is useful with respect to satellite-based positioning, of course. Again, the difficulty is to compare an existing radio system having tremendous merits - GPS - to a future inertial system - TIP. One has to focus on inherent differences and advantages. Fig 12 shows a selection of points. With respect to technical features the considerable differences are apparent. While GPS uses 21+3 satellites, i.e. complex space and control segments for operation of the satellites, TIP requires nothing like that but utilises the accurately known motion data of earth. TIP will be suitable to provide any accuracy required under any environmental condition. Problems connected to very complex radio systems, like multipath, shadowing, and integrity, just don't exist.

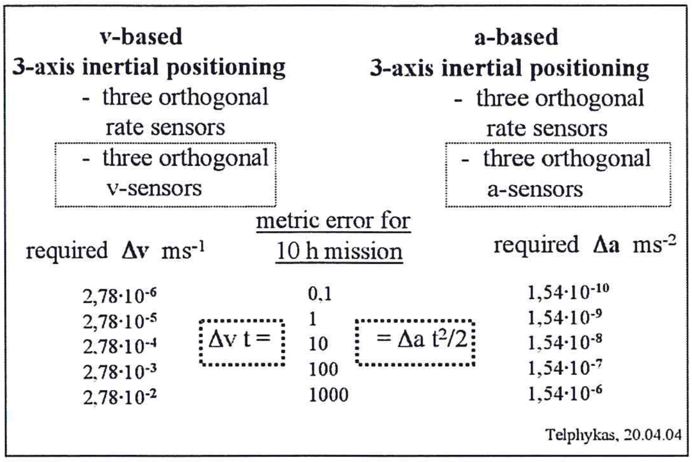

Figure 11: Velocity vs. Acceleration

TIP	GPS/Galileo
Technical	**Technical**
-- no radio signals	-- radio system
-- no environment restrictions	-- environment restrictions
-- no external segments	-- space and contral segments
-- no differential mode needed	-- DGPS for high accuracy
Operational	**Operational**
-- no fees	-- fees for Galileo
-- no central control	-- central control
-- no restrictions	-- P, C/A, S/A
-- global communication needed	-- OS, CS, SoL, PRS, SAR
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Figure 12: Comparison of TIP to GPS/Galileo

With respect to operational aspects it seems likely that Galileo will not be usable without fees, in view of the high cost of operation of such a system even more complex than GPS. While GPS, operated by the Pentagon, can be used, at least partly, without any fee, highest precision is available to the public not at all, or at extra cost like in the case of DGPS. And there is always the risk for civil GPS users that the service is denied to them by the Pentagon under certain conditions and circumstances. This problem may not exist for Galileo to the same extent, once it becomes operational, due to the five services anticipated (Open Service OS, Commercial Service CS, Safety of Life SoL, Public Regulated Service PRS, Search and Rescue SAR). But the price of fees has to be paid in order to earn a return for the private capital anticipated for making Galileo happen.

The risk of false data does not exist for TIP. This inertial system could utilise a unique feature of self calibration based upon the exactly known motion data of earth which cannot be changed by anybody. Every point on earth is characterised by a set of velocity data over time (UTC) and local time, which both can be made available within the local system. The deviation of the *measured* velocities from the real velocities of the position taken, available locally as stored data, can be used for self calibration. Thus there is no drift growing with mission duration as happens with conventional inertial systems. Of course, when using GPS, such self calibration is also possible. However, there is no guarantee that the "real" GPS data are definitely the real ones, not to forget possible complex system errors.

CONCLUSIONS

The Inertial Doppler Effect IDE derived from the classic Sagnac effect opens the gate to a new TIP class of inertial positioning and navigation equipment which utilises earth as the most reliable source of motion data. Such equipment would have the potential to penetrate low-cost mass markets, but as well the potential to meet ambitious demand for highest precision applications for airborne surveying, defence, and space, offering a better and more economic alternative to conventional inertial equipment, and to satellite-based systems as well.

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