Autonomous Time-based Precision Global Positioning

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Abstract

Three-dimensional time-based global positioning by exclusive autonomous precision time measurements is achieved by using astronomical data and a precision clock. The clock is providing the continuously changing universal time t_x (UT), while fixed points in time t_i provide "inertial" references. A local position coordinate, say meridian, is determined by using the equation $L = 360° (t_x - t_i)/T_i$, with T_i being 24 hours, i.e. the time of a daily rotation of earth.

Introduction

Measurement of physical quantities generally requires a reference quantity and a quantity to be measured of the same kind, for instance two distances, two angles, two forces, two frequencies or two points in time. For the autonomous, or self-contained, measurement of two quantities, take angles like geographic longitudes, the reference quantity is to be available at the same location as the quantity to be measured, and it must be independent of the local position. Therefore it has to be ensured that any position change results in a change of the quantity to be measured, but does not effect the reference quantity. Inertially viewed this quantity must be independent of local position, while it changes apparently if the local position is used as the reference. A positioning result, however, is not influenced by the choice of the respective reference frame.

Autonomous positioning, i.e. positioning without any link to the "outside" world, is at present available only via inertial platforms comprising gyros and accelerometers Such platforms provide reference frames calibrated at known positions, while position changes with respect to these positions are measured via three-axis accelerations and twice the integration of them over time[1].

Another well-known example for dynamic effects is the pendulum of Foucault, against which earth rotates at 15°/h. The local observer may interprete this rotation as that of the oscillation plane of the pendulum against earth considered to be fixed, i.e. not rotating. The main disadvantage of such dynamic approaches, i.e. those using forces, are the many error sources, which cause drift errors growing with mission duration. This results in quite a technical effort to limit these errors, while accuracies available still cannot match those achievable via kinematic approaches like GPS.

GPS is based upon differential travel time measurements of signals transmitted from satellites, the positions of which are exactly known at any time and at any receiver via the received signals. GPS signals for public use do not provide the highest possible accuracies which are available to military users only.

Higher accuracies for civil applications are provided by utilization of DGPS, where a ground station transmits corrective signals which enable a civil GPS receiver to reduce the errors of its determined GPS position fix [2,3,4].

For autonomous precision positioning utilization of time signals, which are the most accurate means available in physics, is a must[13]. Time signals for reference and measurement also ensure GPS accuracy, although not self-contained or autonomously.

The objective of this paper is to offer a new time-based kinematic approach for autonomous global positioning without any radio signals and without any gyros and accelerometers, while providing equal and better accuracies than the best GPS or - perhaps in the future - Galileo system can provide for, at a fraction of the expenses required for satellite-based positioning systems. Main issue besides unmatchable accuracy is always the autonomy of the positioning device, i.e. reference and measuring signals are to be available at the same local position. Time-based autonomous positioning, which does not require - exept for calibration at a known position before a mission begins - any external means, is exclusively utilizing differences in time between known fixed points in time and a local precision clock. However, it has to be recognized that the positioning results achieved autonomously a priori do not involve inertial attitude or vectorial information and may be compared in this respect with GPS fixes which also lack such kind of information. In addition, it has to be kept in mind, that also precision clocks are subject to drift[12]. However, the drift error effect can be easily overcome, as will be outlined later.

Physical Background

Just the time signal (UT) of a clock operated autonomously cannot indicate a position. Classic positioning on ships therefore required, in addition to UT, the momentaneous position of the sun, i.e. an inertial reference to determine local time (LT), i.e. the local meridian.

An autonomous positioning approach can substitute the external - inertial - sun by an internal sun, which is, however, no more inertial. The position of this internal sun can be defined with respect to any surface point on earth as a specific time difference, which changes with the change of position of this surface point. Utilization of an internal sun via given known points in local time, accurately related to known points on earth as references, and the variable local time of the own, still unknown position makes it possible, by means of the local precision clock, to continuously determine the local position via local measurements of time differences. This approach applies to the determination of the local meridian, the local parallel, and the local altitude, as will be discussed in more detail later.

At sub-relativistic velocities the two terms universal time (UT) and local time (LT), or local angle (longitude, latitude), are linked by the third term angular velocity ω. For mathematical treatment of different points on earth consideration of the relativity of simultaneity is sufficient. Local time is an analogy to space time, known from General Relativity. Its unit is the second, if position is determined by angles as fractions of 2π, i.e. without metric units.

Exclusive utilization of time for autonomous global positioning in combination with known positions, lines, or planes recommends a careful consideration of what is meant by the term "time". There are many publications on this item [5,6,7,8,9,10]. Of major importance is the term relativity of simultaneity. Also consideration of the Sagnac effect is helpful. In general, it has to be analyzed how different terms like time, clock time, local time, differential time, space time etc. apply to the task of autonomous positioning on earth, and how they can be utilized.

Definitions:

Time: Basic term to describe matter motion
Clock time: Point in time
Point in time: Momentaneous clock display
Differential time: Difference between two points in time
Space time: Desciption term for events
Local time: point in time of a surface point on earth, referred to sun position
Solar time: Based upon a day with 86400 s
Siderial time: Unit is siderial day between two upper culmination points of the spring point V
Time equation: True solar time minus mean solar time
Relativity of simultaneity:

$$ct_2 = (ct_1 - (v/c)x_1)\,(1- v^2/c^2)^{-1/2}$$

For autonomous positioning on earth the relativistic term of the equation above can be neglected since $v^2/c^2 \approx 10^{-8}$. Thus $ct_2 = ct_1 - (v/c)x_1$. The approach described in this paper utilizes time and position differences measured and computed autonomously within a moving object.

It has to be checked which differences result when the measurements are referred to the resting inertial frame instead of to the moving object, and vice versa.

P. Mittelstaedt[3] writes: "If one considers two events $E_1(x_1,t_1)$ and $E_2(x_2,t_2)$, then their space-time (four-dimensional) distance $\Delta(x,t)^2 = c^2(t_2-t_1)^2 - (x_2-x_1)^2$ is invariant towards Lorentz transformations and therefore equal within any inertial frame, the time difference $\Delta t = t_2 - t_1$ itself, however, is not Lorentz-invariant". From this, under certain conditions, the relativity of simultaneity results. It governs the synchronisation of clocks within an inertial system via electromagnetic signals from the spatial origin according to the equation $t = (\Delta s/c) + t_0$, or $t-t_0 = \Delta s/c$ [8]. Δs is the spatial Euklidian distance to the spatial origin, t_0 is the point in time of transmission of a synchronising signal, and c is the velocity of light. It becomes apparent that this equation also results from the application of the Sagnac effect to the same situation.

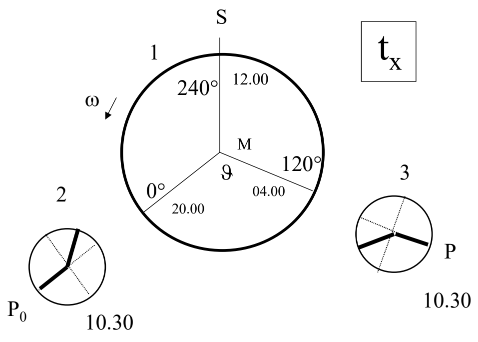
For the Sagnac effect can be formally applied to the rotating earth which transports any fixed electronic clock with its known angular velocity. The well known formula for this effect is $\Delta\Phi = (4A/\lambda r)\beta = (4A/\lambda c)\omega$. With $\Delta\Phi = \omega\Delta t$ it can be derived for earth - the angular velocity ω of which is practically constant and exactly known - $\Delta t = (r/c)\ \Delta\Phi$. The phase $\Delta\Phi$ of any point on a parallel is, in addition, $\omega\Delta t_{LT} = \Delta\Phi$. This leads to $\Delta t\cdot c/r = \Delta t\cdot\Omega = \omega\ \Delta t_{LT}$. The time shift Δt is then the fraction ω/Ω of the local time difference Δt_{LT}. The factor ω/Ω is about $1{,}55\cdot 10^{-6}$. That means travel time for a full circulation of a signal around earth from and to its origin is 0,133673 seconds, compared to the 24 hours of a daily rotation of earth.

These considerations result in a new approach for autonomous time-based positioning on the rotating earth. Fixed points on earth could be determined - if their angular velocities ω_i were known - by simple differential time measurements. For objects moving relative to earth or selected points on it, however, the angular velocity is not known. In these cases one has to use a virtual angular velocity $\Omega = c/r = 47\ s^{-1}$, which results from the diameter of earth 2r and light velocity c. There is no signal which can travel around earth faster than about 133 milliseconds. Compared to the real angular velocity of the daily earth rotation which is $\omega = 7{,}272\cdot 10^{-5}\ s^{-1}$ this means an increase of angular velocity by the factor of about 648 000. The meaning of this factor will be outlined later. Working with the virtual angular velocity Ω is always required for autonomous time-based positioning when the motion of an object with respect to earth surface changes the inertial angular velocity of the object from ω of earth to ω±Δω, with Δω a priori unknown. One then has to continuously utilize the macroscopic Sagnac effect by relying upon the virtual angular velocity Ω. Of course, this approach can also be used for positioning of fixed objects. The introduction of r, however, provides an additional parameter for the respective calculations, which have to consider the variations of r.

The space-time distance Δ(x,t) between two events can be expressed, under certain conditions, by time differences, if one eliminates the constant factors like light velocity c for the spatial terms. It is then possible to determine a spatial coordinate by the local measurement of points in time (UT), while other required points in time can be taken from a local data bank, which contains well-kown astronomical data of the orbits of all points on earth in form of rectascension and declination values.

When determining the local meridian of an object P, a reference point P_0 on a reference meridian which has the angle Φ_x referred to the position of the sun or its local time Φ_x/ω with respect to the line sun-earth center (S-M) at any UT point in time t_x, is always exactly known as a means to determine the still unknown local meridian of object P. The local meridian L (ϑ) results, if P_0 is defined as a point on the Greenwich meridan (0°). Astronomical data of earth on its orbit are needed and are available as published tables[11] of rectascension and declination values.

Fig. 1 Time and Position on rotating Earth

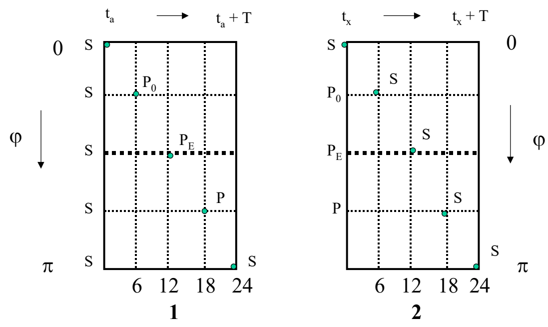


Decisive for the feasibility of the described approach is the utilization of accurate table values in combination with a precision clock, which should not only provide accurate universal time, but also high phase stability and little drift. In addition, for each mission the system is to be calibrated at a known position, for instance a depart point with also known altitude. Another key element is a motion line which represents the angular velocity Ω and directly connects both the unknown local point P und the reference point P_0 .

Measuring Principles

The proposed positioning method is described using twelve figures. In fig.1 the basic situation for autonomous time-based positioning on the rotating earth is outlined. There is sketched a parallel 1 with

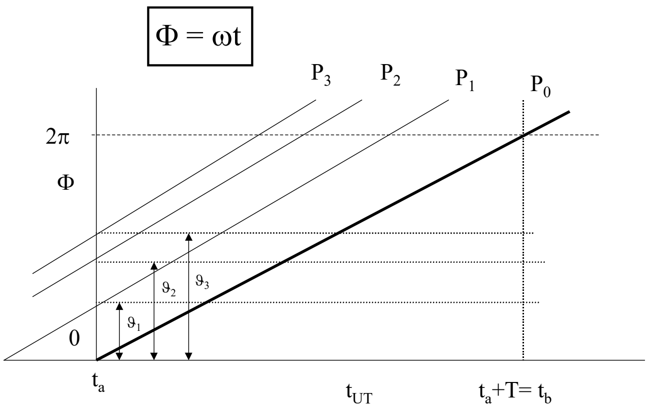
Fig. 2 Inertial and Local Situation



the

Greenwich meridian 0° at 20.00 hours local time and the two meridians 120° with the local time 04.00 hours and 240° with the local time 12.00 hours. Universal time t_x be 10.30 hours. For this configuration there are - sequentially - always changing points in universal time t_{xi} which can be taken from available tables[11]. Clock 2 is located at the Greenwich meridian, with t_x = 10.30 hours, clock 3 is located at the meridian 120°, with the same point in UT time 10.30 hours, however with a different local time, namely shifted by 8 hours to 04.00 hours. Here it becomes clear that t_x is valid for all meridians on earth, while local time is specific for each meridian. It is also apparent that local time and local angle are quantities with identical information concerning position. They are linked by the equation $\Delta t_{LT}\ \omega = \Phi_L$. On earth eight hours difference in local time mean an angle difference of 120°. If now the meridian P of clock 3 is unknown and only t_x (10.30) is available, then this is not sufficient for positioning, but additionally local time has to be introduced. Inertially viewed, the tables[11] already mentioned contain the local time for every meridian at every point in universal time. Locally, however, this does not help directly, and initially one does not know at P that the local meridian is 120°. If one wants to determine the local meridian L (or ϑ) then two quantities are required locally, namely one which does not vary with position, i.e. a reference, and another one which is correctly related to the local time of P. Dynamic methods, operating with gyros and accelerometers, offer these informations. Purely kinematic autonomous methods, however, cause concerns since it is argued that any position change from P_0 to P by the angle ϑ does change the reference time signal in the same way as the measurement signal, and that the angle ϑ of P, which is to be measured with respect to P_0, is thus eliminated. These concerns referring to traditional considerations are understandable and principally justified. However, they do not apply to the time-based positioning method described in this paper, since this approach utilizes fixed points t_{ai} in universal time, which define certain reference meridians with known properties like 00.00 or 12.00 hours local time, and which are neither time- nor position-dependent, i.e. valid for any unknown point P.

Fig. 3 Motion Lines of Points on Earth



During a measurement the time difference between the fixed reference point in time t_a (S) and the continuously changing point in time t_x (UT) of the local position (P) is related to the real position of the sun (S). The time difference measured between a fixed point in time t_a and t_x is a measure for the angular distance of the local meridian to the inertial reference point, the sun. The local time $t_{LT} = \Phi_L/\omega$ of every point on earth surface follows an exactly definable specific function of time t_{UT}. The differences in local time of these practically linear functions can be interpreted as phase differences with respect to the inertial reference, the line sun-earth (S-M). The task is to determine a specific phase difference between P_0 and P.

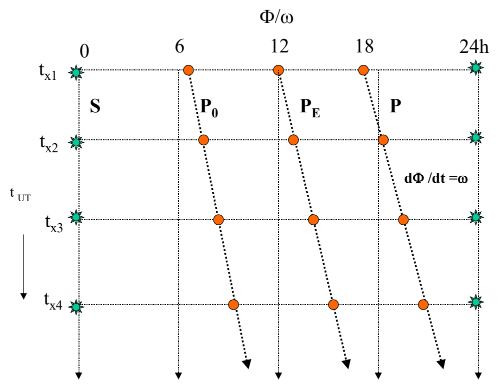
Traditional Considerations

The sketches in fig. 2 indicate the equator as their horizontal center lines, while the respective left ordinates are the reference meridians defined by t_a. The sketches show both principal ways to describe local times, namely as inertial (1) or local (2) situation. One may use either a simulated quasi-inertial display, for which a selected fixed point like the sun is shown fixed also on the display (1), while the simulated earth rotates. Or one choses a fixed earth, with the sun rotating around it (2). If one assumes (in 1) that the (internal) sun is the fixed reference then points P_0, P_E and P move time-dependent with respect to the fixed point in time t_a, which defines a reference meridian pointing to the sun. Or one selects (in 2) a point of the respective three ones as a (local) reference, against which, if this point is moving, the sun seems to move. If there is an observer at point P, the reference point P_0 at the point in time t_a moves, together with the sun, to a different place than if the observer would be at point P_E. The spatial transition from P to P_E would mean a position transformation, which would have to include a respective transformation of local time.

Staying at one of the moving points, the reference point in time t_a can be interpreted as inertial point in the time domain which defines the total local time domain. If locally observed from P, this local time domain - comprising 24 hours or 86400 seconds - would pass this point because of the motion of P with earth. The respective inertial position change of P can be described by a respective change of time or time difference. Of course, there is no true vectorial inertial reference, as will be outlined later. However, the angle difference between P and the selected reference, sun or Greenwhich meridian, exactly fits.

In fig. 3 time dependence of the different local times of points P_0, P_1, P_2 and P_3 is shown. The local times are expressed as local angles. The four points may be located on the same parallels or on different ones. P_0 be the 0°-meridian, which at the point in time t_a exactly points to the sun (12.00). During 24 hours, the period T_L, this meridian rotates by the angle 2π, which is covered at the point in time $t_b = t_a + T_L$. The meridians of the three other points P_1, P_2 und P_3 also cover 2π during $T_L = t_b - t_a$. With respect to the sun, and to P_0, however, these points have the different angle components ϑ_1, ϑ_2, and ϑ_3, which are identical with the respective inertial position angles. If both a local reference signal as well as the measuring signal would contain the respective ϑ, then this quasi-dc component would be eliminated and one could not measure locally and autonomously any change of position. A position change can only be measured if the local reference signal would not contain any position-dependent "dc" angle component.

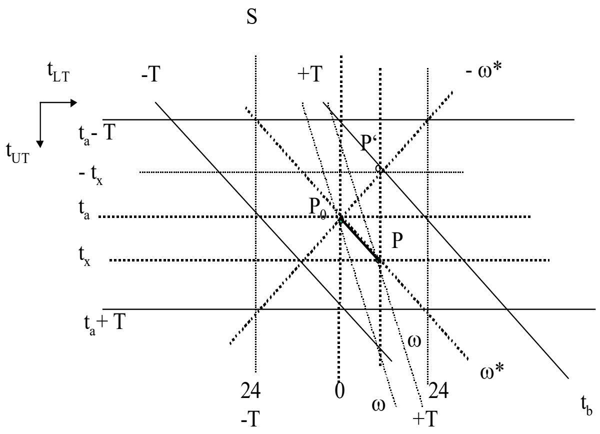
Fig 4 Relation between Universal and Local Time 1



The motion lines of the four points are parallel to each other, and they intersect with the UT axis at specific points in time. For P_0 the intersection point in time is t_a.

The motion lines, the slope of which corresponds to the angular velocity ω of the daily earth rotation, can be considered as inertial motion paths of the points and comprise the elements universal time and local time. A clock in Berlin and another one in Bonn

Fig. 5 Relation between Time and Local Time 2



show identical universal times, but their locations correlate to different local times, of course. The respective local times are usually not available via these clocks, though.

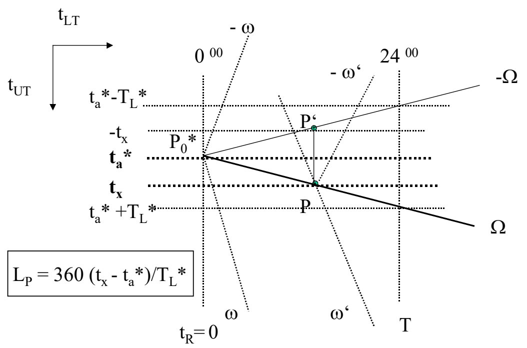
New Approach

Fig.4 shows the general relation of time and meridian or local time of three selected fixed points on earth, the reference point P_0, the calibration point P_E, and the local position P. The figure illustrates the inertial angular change of the three points as a function of time (UT). On earth this is determined by the angular velocity ω of the daily rotation of earth which is practically constant.

The rotation angle is given by the equation $\Delta\Phi = \omega\Delta t$. This equation determines the relation between time and inertial position, i.e. local time, which then is defined by $\Phi/\omega = t_{LT}$. This means, that the universal time difference Δt_{UT} and the local time difference Δt_{LT} are equal, and also that the angular change of any fixed point on earth over time leads to the constant angular velocity ω. However, if one locally connects the unknown point P with a reference point P_0 then the angular velocity of this motion line is not known.That is also valid for points moving relative to earth surface. One has to utilize then, in analogy to the "microscopic" Sagnac effect, the "macroscopic" Sagnac effect with the ultimate angular velocity $\Omega = c/r$. That was already discussed in the introduction section. How the macroscopic Sagnac effect is applied to the determination of local meridians, is dealt with using figures 5 and 6. The same basic procedure is then applied to the autonomous determination of latitude and altitude.

In fig.5 the relation between universal time t_{UT} and local time t_{LT} is explained in more detail. This figure shows the fundamental procedure to determine the unknown local meridian of any fixed point P on earth. In order to make this "inertial" illustration possible the meridian of the reference point P_0 at the point in time t_a has to be known. P_0 changes its inertial position with respect to the inertial reference point sun in an exactly known manner as a function of time on the sketched motion line ω. At the point in time t_a the real P_0 meridian points exactly to the sun, and P_0 at the display, quasi as a sun substitute, may be considered to be an internal sun. In an inertial representation both points P_0 and P - if they are fixed points - change their inertial positions as exactly known functions of time, since they must move with the angular velocity of the daily earth rotation and with respect to the inertial reference point in time t_a. However, the initial location of P at t_a is not known. Therefore only the inertial meridian of P_0 is defined exactly at every point in time. The spatial attitude of any display does not matter at all, since the relative positions are only functions of time. This is valid also, if the local observer at P does not know the local meridian. The time-based determination of a coordinate like a meridian can be based upon the measurement of the time difference

Fig. 6 Principle of Positioning of any Object on Earth



$\Delta t = (t_x - t_a)$, provided the motion line ω* is used which directly connects P and P_0. $\Delta t \cdot \omega^*$ delivers the angular distance between the reference point in time (the internal sun) and the local meridian at any local point in time. For a correct utilization of this basic configuration the following procedure has to be considered. Each point on earth moves on its own individual inertial path. All motion paths are parallel to each other. If one keeps the fixed (inertial) time frame t_a and t_a +T, then the meridian of P_0 changes its position within this time frame or window continuously, while the meridian of P with respect to

P_0 does not change. The total meridan collective moves through this fixed time window once within T (24 hours). Such observation cannot be influenced by the attitude of the used display. But, of course, such a display has no true inertial meaning (which applies to any GPS measurement result, too).

The continuous measurement of the unknown local meridian of P via the clock at this position principally is achieved as follows.

With P_0, the meridian of which - at the point in time t_a - has the local time 12.00 hours and points exactly to the sun, one defines a reference coordinate system, the ordinate of which is the universal time axis and the horizontal axis of which is the local time axis. The inertial motion path ω of P_0 does not meet P, but the motion path ω^* does and connects P_0 and P. If one reflects P at the t_a-axis, one gets point P' which is mirror-symmetric to the t_a-axis. P' sits on a motion line of P_0 which corresponds to $-\omega^*$. The local meridian of P basically results from the equation $L = 360° (t_x - t_a)/T$ provided one knows ω^*.

Physically one may interprete this procedure as an expanded Sagnac approach. Starting at t_a (UT) and 12.00 (LT) from P_0 (internal sun), the stable electronic oscillation within a clock is transported in reality (counterclockwise) to P, which at the moment of measurement t_x (UT) has the local time t_x - 12.00, and additionally a virtual electronic oscillation is transported clockwise to point P'. The time distance (difference) between these two points is twice the time difference t_x-t_a, and unambiguously corresponds to the local meridian of P. Of course, the virtual point P' is not a must for the procedure. But it helps to reduce clock drift.

Since the angular velocity ω^* of P is not known, the procedure sketched in fig. 5 has to be modified. Fig. 6 shows it for fixed as well as for moving objects for meridian measurement. The essential difference to the procedure shown in fig. 5 is the introduction of the known angular velocity $\Omega = c/r$. It is required, since the inertial angular velocity $\omega^* = \omega + \Delta\omega$ of an object on earth with respect to an inertial reference point is not known a priori. $\Delta\omega$ can have any value, except one which is - because of the relativity of simultanenity - beyond c/r.

If one therefore uses this limit of angular velocities on earth, then one don't need to know $\omega^* = \omega + \Delta\omega$. The price to pay is r, the radius of earth which is a variable term and introduces an additional difficulty in autonomous time-based positioning.

Neglecting this, however, determination of the local meridian of a resting or moving P on earth is achieved as follows.

If point P moves relative to earth, then there are various motion lines, which means that for identical points in universal time t_x there are undefined points in local time for P, i.e. local meridians. The direct utilization of $\Delta t = t_x - t_a$ would lead to an error, since the rotation period of P changes from T (86400 seconds) to T', which may be shorter or longer than T, coresponding to the velocity vector of P relative to earth surface.

In order to avoid this effect, the real but unknown angular velocity ω^* of P with respect to P_0 is substituted by the ultimate angular velocity $\Omega = c/r$, as discussed already before. Since for both approaches - Sagnac effect and relativity of simultanenity - the same local meridan must result it follows $\Phi_x = (t_x - t_a)\,\omega' = (t_x - t_a^*)\,\Omega$. In this equation t_a^* can be derived from t_a, as will be discussed later. The time difference $\Delta t^* = t_x - t_a^*$ leads to the looked-for local meridian in connection with the universal time period T* by multiplication with Ω as follows.

Beginning at the - unknown - local position P one draws a motion line Ω with known slope at every point in time t_x until it intersects with the t_a^* - axis. This intersection corresponds to P_0, or the internal sun. The t_a^* - axis is derived from the t_a-axis of fig. 5 by applying the factor ω/Ω to the time difference $t_x - t_a$, measured continuously. That means one derives $t_a^* = (t_x - t_a)\,\omega/\Omega$ and thus can divide $(t_a + T) - t_a$, i.e. T, into Ω/ω ($\approx$ 648 000) ranges of T*. t_a^* is that constant local time axis, which comes closest to the local time axis defined by t_a if one starts at t_x.

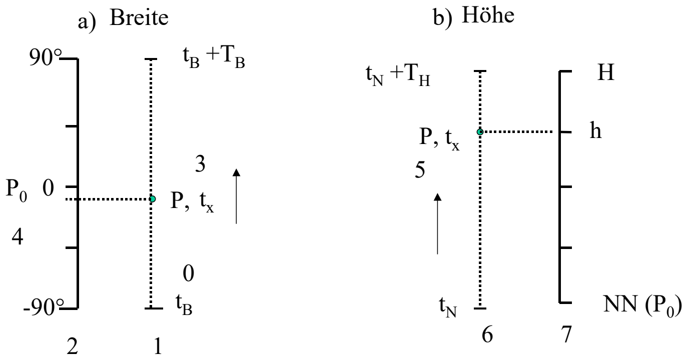
Once P_0^* is determined, one can draw also the motion line $-\Omega$ and define point P' which is reflected at the t_a^*-axis. The distance P-P' , which corresponds to the time difference $2(t_x-t_a^*)$, unambiguoulsy determines the local meridian if P_0 is defined as a point located on the Greenwich meridian, which means that t_a has to be that point in time, at which the Greenwich meridian exactly points to the sun. The local meridian of P is given by $L = K (t_x - t_a^*)/T_L^*$. K is a known factor, which determines the measuring range, i.e. 360° or a derivation from this if one uses "eastern longitude" or "longitude west".

Time resolution of a 24 h period, i.e. one full daily rotation of earth, also determines the achievable angle or local time resolution. If one divides this full rotation cycle or period into 10^6 parts, then one part has the duration of 86,4 ms. During this time any point at the equator moves tangentially with respect to the earth center by about 40 m. There is no need to also consider orbit velocity.

Within the local time period T_{LT} one has the assumed 10^6 time intervals of the universal time range as also local time or angle segments. One angle segment corresponds to $2\pi/10^6 = 1{,}59 \cdot 10^{-5}$ or

20,63". Since Ω is employed, which has a period duration of $T^* = 0{,}133$ s, the assumed resolution of 10^6 causes clock impulse intervals of $0{,}133 \cdot 10^{-6}$ s. This corresponds to a clock frequency of about 7 Mhz. Since the present clock rate state of art is three orders of magnitude higher, resolution can be

Fig. 7 Principle of autonomous Measurement of Latitude and Altitude



increased respectively, which means to go from the assumed 40 m (10^6) resolution to 0,04 m (10^9). Potential resolution limits are not imposed by the achievable clock rate, but by the drift rate of the applied clock. Also atomic clocks are subject to drift[12]. It is, however, possible to neutralize any clock drift by employing measuring time intervals of more than one period, for instance by using instead of one daily rotation a yearly rotation which means 365 daily rotations.

Points in time as well as angles usually are already coded as digital information. While local time is periodic with 24 hours, universal time changes unidirectionally. Since t_a and the meridian pointing to the sun at this point in time are very acurately known, as well as $t_a \pm nT$ (n may be any integer), and also t_x can be measured with utmost accuracy, the local meridian derived from these accurate terms - which practically can't be distorted - is also highly accurate.

The reference point in time t_a - rectascension - is very accurately known from literature[11]. Despite being an inertial angle, it is given as an accurate time for every day of a year. The values of all meridians could be stored for all days of a year autonomously, and be used as needed for positioning computations.

The point in time t_a^* is a numerical value for an inertial reference. But as a number it only defines which angle Φ_x a chosen reference meridian like that of Greewich has at any point in time t_x from the selected inertial reference (sun). Locally there is no true, vectorial inertial information connected to these numbers. The local meridian of P is correctly determined, but the inertial attidude of this meridian in space remains unknown from only this information.

The option to shift the fixed point in time t_a^* by n periods - with n being any integer, for instance 365 for going to the yearly instead of to the daily rotation of earth - opens two ways to improved performance. The first one leads to the reduction of clock rate, and the second one allows for neutralizing drift rates of clocks, which means that simpler clocks can substitute high performance atomic clocks.

When using the numerical time for one yearly earth rotation, the required clock rate for the chosen position resolution can be reduced by the factor 365. And the position error caused by the clock drift is reduced then by the same factor.

Latitude and Altitude

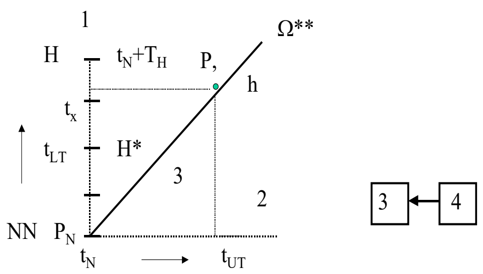
Fig. 7 shows principally how latitude and altitude can be measured autonomously. Fg. 7a shows time range 1 and local time range 2, which is drawn thicker. At a point in time t_x (UT) the point P with its locally unknown position is assumed to be located at a southern parallel of about 10°. For measurement the local view is used and compared with the simulated inertial view, similarly to what was described already for meridian measurement using figures 5 and 6. Also for latitude measurement an inertial reference point P_B has to be introduced, which is determined by a fixed point in time t_c. The difference in time $t_x - t_c = \Delta t_B$ corresponds to the parallel of P within the latitude range of ± 90° and is given by $B = \pm 90° \, (\Delta t_B / T_B)$. For defining the inertial reference point P_B declination can be used which is available very accurately from literature[11] and which changes daily during the yearly orbit of earth around the sun.

Fig. 7b illustrates the principle of the autonomous time-based measurement of altitude. As inertial reference sea level NN is chosen, although the real sea level locally changes as a function of tides and weather. It is possible to use a special internal scanning frequency for the altitude range 6 from NN to H. Because of moving objects like aircraft again it is required to introduce the highest possible virtual scanning velocity $c/(H\text{-}NN) = \Omega^*$ or $\Omega^* = c/H^*$. The altitude of P then can be derived, in analogy to longitude and latitude determination, from $h = H^*(t_x - t_e)/T_H$, as is discussed in more detail referring to fig. 8.

This additional discussion is useful since it discloses that autonomous time-based determination of a quantity does not require to consider earth rotation. And it clarifies that, like altitude, any distance to selected time-based reference points can be measured autonomously. It should be emphasized again, however, that the true inertial attitude of the altitude range H* cannot be taken from the scalar measurement results. One would have to get the plumb vector from a different sensor.

Fig. 8 explains hwo the time-based autonomous altitude determination of P within a linear range H* is achieved. 1 is the time range, H* the altitude range from sea level NN to maximum altitude H, t_c the reference point in time when the scanning frequency - which simulates a relative motion between an inertial reference system and the actual location of P - starts at sea level (NN).

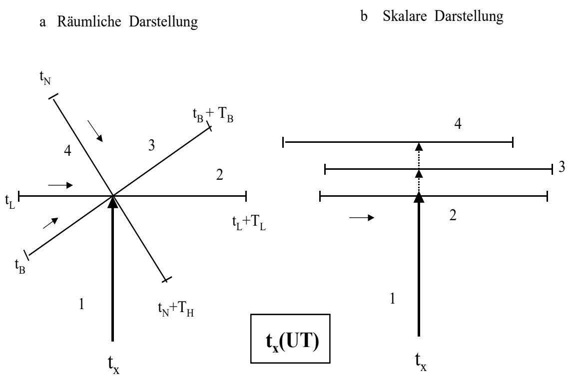
Fig. 8 Autonomous Measurement of Altitude h



$$h = H^* (t_x - t_N)/T_H$$

It is assumed that the reference level P_N periodically scans the altitude range H* as a function of universal time t_{UT} with the angular velocity Ω** = c/H*. Thus a common virtual motion line Ω** is allocated to P_0 and P.

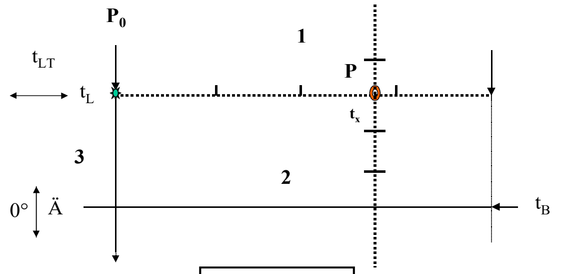
Fig. 9 Time Signals of autonomous 3D-Positioning



At P one measures universal time t_x and thus has the time difference t_x - t_N at every point in universal time. Because of the selected angular velocity this time difference is directly proportional to the altitude of P. It helps in understanding to consider this approach as a kind of autonomous on-way-DME where a reference point in time t_N is allocated to the "inertial" sea level. If the altitude of P remains unchanged, then also the respective time difference as a phase difference between the periodic reference signal with the period T_H and universal time t_x remains unchanged. Relativity of simultaneity makes this approach possible.

Once again it is emphasized that the measured altitude has no vectorial character.

Fig. 10 Autonomous time.based Positioning (Longitude and Latitude)



$$L = 360 (t_x - t_L)/T_L$$

$$B = \pm 90° (t_x - t_B)/ T_B$$

Measurement of altitude h within the altitude range H* is achieved as follows. Beginning at sea level which is marked with the fixed reference point in time t_N P_N scans virtually the altitude range H* periodically with the angular velocity Ω* = c/H*.

During the scan of H* the reference signal P_B (NN) coincides at the point in time t_x with the local point P = H* (t_x - t_N) /T_H. If P is at the low end of H*, i.e. at sea level and at the start of a scan period T_H then t_x equals t_N, i.e. t_x - t_N = 0, and therefore P = NN. One could say that the phase difference of P referred to sea level is zero. If P is located in the middle of H*, then t_x - t_N = T_H/2 and therefore P = H*/2. One could say, the phase difference between P and NN is π. If P is at the high end of H* then t_x - t_N = T_H , and thus P = H. The phase difference between P and NN is then 2π.

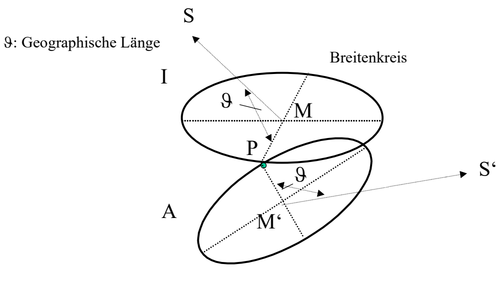
This procedure is feasible since the position of P within H* can be changed only at the expense of local time, i.e. differential phase, while the reference point in time t_N is not effected by any positional change of P. Therefore a change of P altitude results in a change of local time and thus a change of phase between P and sea level which allows for measuring such altitude change. Graphically that was explained already in figures 5 and 6 for meridian measurement.

The result of these explanations are the given equations which allow for continuous rapid computations of the respective 3D-position of P at any point in time. The autonomous altitude measurement (and any other distance measurement) can be achieved locally via a precise clock 3 and a computer 4.

Autonomous v. Inertial

Fig. 9 illustrates how the three-dimensional positioning results in time differences 2 (L), 3 (B) and 4 (h) can be interpreted, which are achieved by only measuring them at a point in time t_x. Fig. 9a shows a pseudo spatial sketch with the time intervals T_i arranged orthogonally to each other for longitude, latitude and altitude measurements. These time intervals are beginning with specific fixed points in universal time t_L for longitude, t_B for latitude and t_N for altitude, and end after the respective periods of time T_L, T_B and T_H. The measured position is determined by the local point in time t_x delivered by the local clock. This clock impulse divides the periods T_i of the three coordinate ranges (2π, $\pm\pi/2$, H) by a ratio corresponding to the respective coordinates of P. The orthogonal arrangement of the coordinates is arbitrary, since the numerical values of the coordinates don't involve any vectorial information. Therefore they can be arranged, as shown in fig. 9b, also as parallel line segments. In practical applications the coordinates may usually not be displayed directly, but processed suitably within a more complex master control system.

Fig. 11 Difference between autonomous (A) and inertial (I) Meridian Determination



In fig. 10 it is shown how two positioning results, longitude and latitude, can be displayed on an x-y-display.

The y-axis 3 shows the latitude of P, the x-axis 2 the longitude of it. P is chosen as the fixed frame while equator and Greewich meridian move with respect to it although in reality P moves. Of course, the available data allow for dislaying a fixed earth with P moving on it as well.

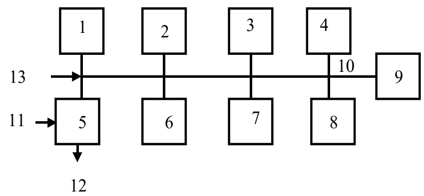
Fig. 11 shows the difference between autonomous and inertial measurements. I is the inertial system with sun S, point P on the real parallel circle with its center M and the longitude ϑ at the point in time t_a, when the Greenwich meridian points exactly to the sun. This inertial system I has to be discriminated from the autonomous system A, the attitude of which in space is not identified. Only point P is common to both systems. All points of system A are numerical, time-dependent values which have no true inertial meaning. But their relative coordinates at any point in time t_x are correct like within a true inertial system.

If one needs the autonomous coordinates as inertial coordinates, then additional information from vectorial sensors like plumb and north gyros is required. The transformation of autonomous quantities into inertial ones is a familiar mathematical exercise. A principal problem for this conversion may arise when the autonomous values are more accurate than the inertial vectors available. In those cases more complex conversion procedures (not discussed here) may help.

Technical Realisation

Fig. 12 gives an idea of how to realize technically the discussed positioning procedures.The block diagram shows the clock 1, the longitude processor 2, the latitude processor 3, the altitude processor 4, the A/I converter 6, the data bank 5 with input 11 and output 12, central control 7, control and display interface 8, and the data bus 10, which connects all subsystems with each other, and the bus control 9 with the remote control input 13. The device works as follows (calibration assumed to have been done already).

Fig. 12 Example for technical Realisation (Block Diagram)



The data bank 5 supplies the three coordinate processors 2 (L), 3 (B) and 4 (h) with the specific fixed points in time t_L t_B and t_H and the respective periodic time intervals T_L, T_B und T_H. Clock 1 supplies the three processors with the universal time t_x resolved as required by the desired positioning resolution. Generally geodetic resolution requirements are much tougher than those for fast vehicles.

The coordinate processors continuously compute - based upon the data input - the three coordinates longitude, latitude and altitude with the refresh rate required by the system, and digitally delivers these values to the master system via the data bus and the output interface 12, and via the converter 6 to the control and display interface 8.

Conclusions

Summarizing, the new autonomous, time-based positioning approach described in this paper for measuring longitude, latitude and altitude combines the signal of a UT clock with very accurate data of fixed points in time and coordinates belonging to it. Positioning is reduced to the measurements of time differences.

The difference between a given fixed point in time t_a and the local clock time t_x divides the respective period time T, and therefore the respecctive measurement range, by the ratio $\Delta t_x /T$, from which the coordinate to be measured can be derived very accurately. The procedure can be used for precise 3D positioning within microseconds independent of the attitude of the object to be positioned.

The described autonomous positioning method offers a number of advantages compared to the state of art, in particular achievable maximum accuracy, full autonomy, high integrity depending mainly upon the local clock and software, and the suitability for application in any fixed and moving objects where a radio system like GPS can't be used. Clock drift rates can be neutralized by selecting long time intervals, like those for one year or more instead of for a day. Calibration essentially is a test of the local clock and does not comprise special expertise.

The described procedure can be supplemented by further functions, for instance autonomous derivation of velocity over ground of an airplane via several position points sequentially measured. In addition, local inertial vectors like north or lumb can be corrected and improved in accuracy.

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